Here the longitudinal wave u_y has a dispersion similar to that of the transverse wave in the previous case with c_t replaced by c_l, and the transverse wave u_x propagates with a phase velocity c_t.

If the field H is large and the angles are close to $\pi/4$, we get from (15) that

$$-2a^2(k/d)\sin\alpha\cdot\cos\alpha u_x+(-\omega^2+2a^2(k/d)\cos^2\alpha)u_z=0;$$
$$(-\omega^2+2a^2(k/d)\sin^2\alpha)u_x-2a^2(k/d)\sin\alpha\cdot\cos\alpha\cdot u_z=0;$$
$$(-\omega^2+2a^2k/d)u_y=0.$$

The wave u_y propagates independently with a group velocity $(a/\sqrt{2\varkappa}\cdot\mathbf{k}/k)$. The dispersion relationship for the waves u_x, u_z

$$\omega^2=2a^2k/d$$

leads to a group velocity $a/\sqrt{2\varkappa}\cdot\mathbf{k}/k$. Thus all three deformations propagate along the vector **k** with an identical velocity U_H independently of the angle of inclination α.

We now give some numerical calculations. The Alfvén velocity is comparable with the velocity of sound in an elastic medium when $H^*\sim\sqrt{E}$ (for steel $E=2\cdot10^6$ kg/cm², when $H^*\approx10^6$ Oe). For a thin film $H^*\sim\sqrt{Ed/\lambda}$, where λ is the wavelength; i.e., when $d\ll\lambda$ the magnetic field begins to affect the deformation at much smaller values of H. Moreover, a conducting layer can be deposited on films of materials with small values of Young's modulus (for example, rubber, polyethylene, and so on), and for these the effects will begin to occur at magnetic fields of the order of a few oersteds.

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, The Theory of Elasticity, 2nd ed., Addison-Wesley (1971).
2. L. D. Landau and E. M. Lifshits, The Electrodynamics of Continuous Media, Addison-Wesley (1960).

SCATTERING AND VELOCITY DISPERSION OF ULTRASONIC WAVES IN POLYCRYSTALS OF ORTHORHOMBIC SYMMETRY

A. A. Usov, A. G. Fokin, and T. D. Shermergor

UDC 534.22

The scattering coefficients and the velocity of propagation of longitudinal and transverse ultrasonic waves in polycrystals of orthorhombic and higher symmetry are computed by the method of renormalization of the equations of motion. The formulas thus obtained are compared with the known asymptotic expressions for long and short waves. A numerical computation carried out for aluminum shows that for $qa\sim1$ (q is the wave number; a is the correlation scale) the power index determining the frequency dependence of the scattering coefficient decreases monotonically from 4 to 2 for the transverse waves, while for the longitudinal waves this dependence is nonmonotonic, i.e., the power index decreases from 4 to 1, after which it increases again to 2. In the Rayleigh region ($q_l a<1$) the scattering coefficient of the longitudinal waves increases with a power index smaller than 4.

A large number of studies has been devoted to the scattering of ultrasonic waves at the inhomogeneity grains of crystals; a review of these studies is given in [1]. The complexity of the computation leads to the re-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 180-190, May-June, 1976. Original article submitted July 2, 1975.

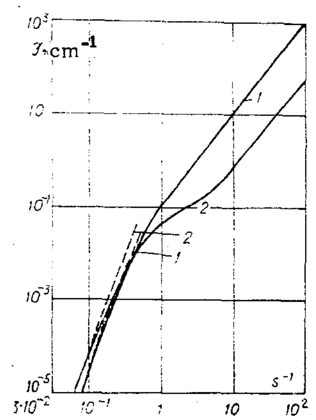

Fig. 1

TABLE 1

Material	$\frac{c_{11}}{\langle c_{11}\rangle}$	$\frac{c_{22}}{\langle c_{11}\rangle}$	$\frac{c_{33}}{\langle c_{11}\rangle}$	$\frac{c_{12}}{\langle c_{12}\rangle}$	$\frac{c_{23}}{\langle c_{12}\rangle}$	$\frac{c_{13}}{\langle c_{12}\rangle}$	$\frac{c_{44}}{\langle c_{44}\rangle}$	$\frac{c_{55}}{\langle c_{44}\rangle}$	$\frac{c_{66}}{\langle c_{44}\rangle}$
Topaz	0,865	1,101	0,905	1,364	0,959	0,905	0,925	1,136	1,119
$MgSO_4\cdot7H_2O$	1,016	0,770	1,196	1,233	0,895	0,891	0,577	1,257	1,198
Mg_2SiO_4	1,330	0,813	1,022	0,749	0,991	1,003	0,809	0,983	0,962
Na tartrate	0,870	1,032	1,255	0,858	1,056	0,960	1,262	0,315	0,997
Seignette salt	0,809	1,036	1,275	0,807	1,008	1,090	1,321	0,325	1,029
HJO_3	0,675	1,212	0,972	1,376	0,636	1,353	1,010	1,209	0,955
Argonite	1,555	0,849	0,820	1,689	0,711	0,076	1,045	0,632	1,037
$SrSO_4$	0,983	0,999	1,211	1,105	0,884	0,864	0,745	1,540	0,149

sult that usually the investigation is restricted to the long-wave and short-wave asymptotic forms and the region for which the wavelength is of the same order as the average size of the grains remains uninvestigated. The object of the present work is to compute the velocity of propagation of ultrasonic waves and their scattering coefficient in the entire range of wavelengths.

1. We shall assume that the symmetry of the polycrystal is not lower than orthorhombic and we carry out the computation with the correlation approximation proposed in [2]. The procedure of computation is discussed in [2, 3], where it is shown that for determining the velocity and the scattering coefficient it is necessary to evaluate the integral

$$I_{pqrs} = \int G_{pr}(\mathbf{r}, \omega)[\varphi(\mathbf{r}) \cos \mathbf{qr}]_{,qs} dV, \tag{1.1}$$

where G_{pr} is the Green's tensor of the wave equation for a medium with averaged properties; $\varphi(\mathbf{r})$ is the coordinate dependence of the binary correlation function of the elasticity tensor, which for the polycrystals is taken in the form $\varphi(\mathbf{r}) = \exp(-r/a)$; $\mathbf{q}$ is the wave vector; a is the correlation scale; and the indices occurring after a comma denote differentiation with respect to the corresponding coordinate.

For evaluating integral (1.1) we make the change of variables $\mathbf{q} = q\mathbf{l}$, $\mathbf{qr} = \xi \equiv \xi\mathbf{n}$ and express the frequency in terms of the wave number with the use of the formula $q = \omega/c$, where c is the phase velocity in the medium with averaged properties. Then integral I_{pqrs} can be written in the form

$$I_{pqrs} \int [n_{pr}g + \delta_{pr}h]\, \xi\, (\partial^2/\partial\xi_q\partial\xi_s)(e^{-\xi s} \cos \xi\mathbf{l})\, d\xi d\Omega_n; \tag{1.2}$$

$$n_{pq\ldots r} \equiv n_p\, n_q \ldots n_r, \quad s \equiv 1/qa, \quad d\Omega_n \equiv d\xi/\xi^2 d\xi, \tag{1.3}$$

where the explicit forms of functions g and h are given, for example, in [2]. Equation (1.2) is valid for longitudinal and transverse waves. In the first case, all the quantities must be expressed in terms of q_l, c_l, ξ_l, s_l, while in the second case they must be expressed through q_t, c_t, ξ_t, s_t. The velocities c_l and c_t are defined in [3]. The functions g and h for the longitudinal and transverse waves are respectively equal to

$$\begin{aligned} 4\pi\rho[c^2\xi^2 g(\xi, c, \varkappa)]_{(l)} &= [(\xi^2 - 3 - 3i\xi)e^{-i\xi} - (\varkappa^2\xi^2 - 3 - 3i\varkappa\xi)e^{-i\varkappa\xi}]_{(l)}, \\ 4\pi\rho[c^2\xi^2 h(\xi, c, \varkappa)]_{(l)} &= [(i\xi + 1)e^{-i\xi} + (\varkappa^2\xi^2 - 1 - i\varkappa\xi)e^{-i\varkappa\xi}]_{(l)}; \end{aligned} \tag{1.4}$$

$$\begin{aligned} 4\pi\rho[c^2\xi^2 g(\xi, c, \varkappa)]_{(t)} &= [(3 - \xi^2 + 3i\xi)_e{}^{-i\xi} + (\varkappa^2\xi^2 - 3 - 3i\varkappa\xi)e^{-i\varkappa\xi})_{(t)}, \\ 4\pi\rho[c^2\xi^2 h(\xi, c, \varkappa)]_{(t)} &= [(\xi^2 - 1 - i\xi)_e{}^{-i\xi} + (i\varkappa\xi + 1)e^{-i\varkappa\xi}]_{(t)}, \end{aligned} \tag{1.5}$$

where the index l enclosed in parentheses denotes that the quantities ξ, c, $\varkappa$, g, h must be replaced by ξ_l, c_l, $\varkappa_l$, g_l, h_l and similarly for the index t:

$$\varkappa_l \equiv c_l/c_t, \quad \varkappa_t \equiv c_t/c_l. \tag{1.6}$$

Comparing expressions (1.4), (1.5) we find that the functions g_t, g_l and h_t, h_l are related through the formulas

$$\begin{gathered} g_t(\xi_t, c_t, \varkappa_t) = -g_l(\xi_t, c_t, \varkappa_t); \\ h_t, (\xi_t, c_t, \varkappa_t) = -h_l(\xi_t, c_t, \varkappa_t) + [(1/4\pi\rho c^2)(e^{-i\xi} + \varkappa^2 e^{-i\varkappa\xi})]_{(t)}, \end{gathered} \tag{1.7}$$

which enable us to evaluate integral (1.2) for the case of transverse waves from its value for the longitudinal waves.

Carrying out differentiation under the integral sign in (1.2) and integrating over the angles, we obtain

$$\begin{gathered} I_{pqrs} = \int g(\xi, c, \varkappa) e^{-\xi s} \{ s[-\delta_{sq} J_{pr} + (1+\xi s) J_{pqrs} + \\ + l_q \xi J_{prs} + l_s \xi J_{prq}] - l_{sq} \xi J_{pr} \} \, d\xi + \int h(\xi, c, \varkappa) e^{-\xi s} \times \\ \times \{ s[-\delta_{pr}\delta_{sq} J + (1+\xi s)\delta_{pr} J_{sq} + l_q \xi \delta_{pr} J'_s + l_s \xi \delta_{pr} J'_q] - l_{sq}\xi\delta_{pr} J \} \, d\xi; \end{gathered} \tag{1.8}$$

$$\begin{gathered} J_{pqrs} = \int n_{pqrs} \cos \xi l d\Omega_n = (4\pi/\xi^5)[l_{pqrs}(\xi^4 \sin\xi + \\ + 10\xi^3 \cos\xi - 45\xi^2 \sin\xi - 105\xi\cos\xi + 105\sin\xi) + \psi_{pqrs}(-\xi^3\cos\xi + \\ + 6\xi^2\sin\xi + 15\xi\cos\xi - 15\sin\xi) + \delta_{pqrs}(-\xi^2\sin\xi - 3\xi\cos\xi + 3\sin\xi)]; \end{gathered} \tag{1.9}$$

$$\begin{gathered} J_{pr} = J_{pqrq} = (4\pi/\xi^3)[l_{pr}(\xi^2\sin\xi + 3\xi\cos\xi - 3\sin\xi) + \delta_{pr}(\sin\xi - \xi\cos\xi)]; \quad J = J_{pp} = (4\pi/\xi)\sin\xi; \\ J_{prs} = \int n_{prs} \sin \xi l d\Omega_n = (4\pi/\xi^4)[l_{prs}(-\xi^3\cos\xi + 6\xi^2\sin\xi + 15\xi\cos\xi - \\ - 15\sin\xi) - (l_p\delta_{rs} + l_r\delta_{ps} + \delta_{pr}l_s)(\xi^2\sin\xi + 3\xi\cos\xi - 3\sin\xi)]; \\ J'_s = J_{pps} = (4\pi/\xi^2) l_s (\sin\xi - \xi\cos\xi); \end{gathered}$$

$$\begin{gathered} l_{pq\ldots r} = l_p l_q \ldots l_r; \quad \delta_{pqrs} = \delta_{pq}\delta_{rs} + \delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr}; \\ \psi_{pqrs} = l_{pq}\delta_{rs} + l_{pr}\delta_{qs} + l_{ps}\delta_{qr} + l_{qr}\delta_{ps} + l_{qs}\delta_{pr} + l_{rs}\delta_{pq}. \end{gathered}$$

We now carry out the integration of (1.8) with respect to ξ. For this purpose we make use of formulas (1.9) and also (1.4) or (1.5) depending on whether the computation is carried out for longitudinal or transverse waves. The substitution leads to the integrals

$$\begin{gathered} J_{ns} = \int_0^\infty e^{-\beta\xi} \xi^{-n} \sin\xi d\xi; \quad J_{nc} = \int_0^\infty e^{-\beta\xi}\xi^{-n}\cos\xi d\xi; \\ j_{ns} = \int_0^\infty e^{-\alpha\xi}\xi^{-n}\sin\xi d\xi; \quad j_{nc} = \int_0^\infty e^{-\alpha\xi}\xi^{-n}\cos\xi d\xi, \end{gathered} \tag{1.10}$$

where

$$\alpha = s + i; \quad \beta = s + i\varkappa. \tag{1.11}$$

After carrying out the computation, we get

$$\begin{gathered} J_{0s} = 1/(1+\beta^2); \quad J_{0c} = \beta/(1+\beta^2); \quad J_{1s} = J_s = \operatorname{arctg}(1/\beta); \\ J_{1c} = J_c; \\ J_{2s} = 1 + J_c - \beta J_s; \quad J_{2c} = \delta - \beta - J_s - \beta J_c; \\ J_{3s} = (1/2)[2\delta - 3\beta - 2\beta J_c + J_s(\beta^2 - 1)]; \\ J_{3c} = (1/4)[2\delta^2 - 4\delta\beta + 3\beta^2 - 3 + 2J_c(\beta^2 - 1) + 4\beta J_s]; \\ J_{4s} = (1/36)[18\delta^2 - 36\delta\beta + 33\beta^2 - 11 + 6J_c(3\beta^2 - 1 + 6\beta J_s(3 - \beta^2)]; \\ J_{4c} = (1/36)[12\delta^3 - 18\delta^2\beta + 18\delta(\beta^2 - 1) + 33\beta - 11\beta^3 + 6\beta J_c(3 - \beta^2) + 6J_s(1 - 3\beta^2)]; \\ J_{5s} = (1/72)[24\delta^3 - 36\delta^2\beta + 12\delta(3\beta^2 - 1) + 25\beta - 25\beta^3 + 12\beta J_c(1 - \beta^2) + 3J_s(1 - 6\beta^2 + \beta^4)]; \\ J_{5c} = (1/288)[72\delta^4 - 96\delta^3\beta + 72\delta^2(\beta^2 - 1) + 48\delta\beta(3 - \beta^2) + \\ + 25 - 150\beta^2 + 25\beta^4 + 12J_c(\beta^4 - 6\beta^2 + 1) + 48\beta J_s(\beta^2 - 1)]; \\ J_{6s} = (1/7200)[1800\delta^4 - 2400\delta^3\beta + 600\delta^2(3\beta^2 - 1) + \\ + 1200\beta\delta(1 - \beta^2) + 685\beta^4 - 1370\beta^2 + 137 + 60J_c(5\beta^4 - 10\beta^2 + 1) + 60\beta J_s(-5 + 10\beta^2 - \beta^4)]; \\ J_{6c} = (1/7200)[1440\delta^5 - 1800\delta^4\beta + 1200\delta^3(\beta^2 - 1) + \\ + 600\delta^2\beta(3 - \beta^2) + 300\delta(\beta^4 - 6\beta^2 + 1) - 685\beta + 1370\beta^3 - \\ - 137\beta^5 + 60\beta J_c(-\beta^4 + 10\beta^2 - 5) + 60J_s(-1 + 10\beta^2 - 5\beta^4)]; \end{gathered} \tag{1.12}$$

$$J_{7s} = (1/21600)[4320\delta^5 - 5400\delta^4\beta + 1200\delta^3(3\beta^2 - 1) + 1800\delta^2\beta(1 - \beta^2) + 180\delta(5\beta^4 - 10\beta^2 + 1) - 441\beta + \\ + 1470\beta^3 - 441\beta^5 + 60\beta J_c(-3\beta^4 + 10\beta^2 - 3) + 30J_s(-1 + 15\beta^2 - 15\beta^4 + \beta^6)].$$

Here $1/\delta$ is an infinitely small quantity which is introduced into the integral (1.10) instead of the lower limit. The integrals with parameter α can be obtained from this integral by a formal substitution of β by α. Formulas (1.12) were evaluated by integration by parts after which the functions $e^{-\beta/\delta}$, $\sin(1/\delta)$, and $\cos(1/\delta)$ were expanded in series.

2. Let us consider the longitudinal waves. For this case the quantities c, q, s, $\varkappa$, β, α, etc., must have the index l (exceptions are only ρ, ω, and a). However, for simplifying writing we shall omit the index. Using integrals (1.12) and expressions (1.4), (1.9), from formulas (1.8) we get

$$I_{pqrs} = l_{pqrs}F_1 + \psi_{pqrs}F_2 + \delta_{pqrs}F_3 + \delta_{pr}l_{qs}F_4 + \delta_{pr}\delta_{qs}F_5; \quad (2.1)$$

$$\rho c^2 F_n \equiv R_n(\beta, \varkappa) - S_n(\alpha) + Q_n(\alpha, \beta, \varkappa). \quad (2.2)$$

where the functions $R_n(\beta, \varkappa)$ are given by equations

$$\begin{aligned}
8R_1(\beta, \varkappa) &= s^2[23\beta^2 + 21\beta^4 - 3\beta J_s(3 + 10\beta^2 + 7\beta^4)] + \\
&+ (1/2)s[(233/5)\beta + (142/3)\beta^3 - 7\beta^5 - J_s(11 + 63\beta^2 + 45\beta^4 - 7\beta^6)] + \\
&+ 12[\beta^2 - \beta J_s(1 + \beta^2)] + i\varkappa s^2[-55\beta - 105\beta^3 + 3J_s(3 + 30\beta^2 + 35\beta^4)] + \\
&+ i\varkappa s[-114/5 - 97\beta^2 + 21\beta^4 + 3\beta J_s(21 + 30\beta^2 - 7\beta^4)] + 12i\varkappa[-3\beta + J_s(1 + \\
&+ 3\beta^2)] - 4\varkappa^2 s^2[2/(1 + \beta^2) + 10/3 + 35\beta^2 - 5\beta J_s(3 + 7\beta^2)] - \varkappa^2 s[- \\
&- 16\beta/(1 + \beta^2) + (305/3)\beta - 35\beta^3 - J_s(21 + 90\beta^2 - 35\beta^4)] - 8\varkappa^2[-1/(1 + \beta^2) + 3 - 3\beta J_s]; \\
8R_2(\beta, \varkappa) &= s^2[-5\beta^2 - 3\beta^4 + 3\beta J_s(1 + 2\beta^2 + \beta^4)] + (1/2)s[-(39/5)\beta - \\
&- (10/3)\beta^3 + \beta^5 + J_s(5 + 9\beta^2 + 3\beta^4 - \beta^6)] + i\varkappa s^2[13\beta + 15\beta^3 - \\
&- 3J_s(1 + 6\beta^2 + 5\beta^4)] + i\varkappa s[32/5 + 7\beta^2 - 3\beta^4 - 3\beta J_s(3 + 2\beta^2 - \\
&- \beta^4)] + 4\varkappa^2 s^2[4/3 + 5\beta^2 - \beta J_s(3 + 5\beta^2)] + \varkappa^2 s[(23/3)\beta - 5\beta^3 - J_s(3 + 6\beta^2 - 5\beta^4)]; \\
40R_3(\beta, \varkappa) &= s^2[9\beta^2 + 3\beta^4 + 8J_c - \beta J_s(15 + 10\beta^2 + 3\beta^4)] + \\
&+ (1/2)s[-(443/15)\beta - (14/3)\beta^3 - \beta^5 - 16\beta J_c + J_s(-5 + 15\beta^2 + 5\beta^4 + \\
&+ \beta^6)] + 5i\varkappa s^2[-3\beta^3 - 5\beta + 3J_s(1 + 2\beta^2 + \beta^4)] + i\varkappa s[184/15 + 9\beta^2 + \\
&+ 3\beta^4 + 8J_c - J_s\beta(15 + 10\beta^2 + 3\beta^4)] - 20\varkappa^2 s^2[2/3 + \beta^2 - \beta J_s(1 + \beta^2)] - \\
&- 5\varkappa^2 s[-\beta^3 - (5/3)\beta + J_s(1 + 2\beta^2 + \beta^4)]; \\
2R_4(\beta, \varkappa) &= s^2[\beta^2 - \beta J_s(1 + \beta^2)] + s[(1/3)\beta - \beta^3 + J_s(\beta^4 - 1)] + \\
&+ [-\beta^2 + J_s\beta(1 + \beta^2)] + i\varkappa s^2[-3\beta + J_s(1 + 3\beta^2)] - 4i\varkappa s[1/3 - \\
&- \beta^2 + \beta^3 J_s) - i\varkappa[-3\beta + J_s(3\beta^2 + 1); \\
6R_5(\beta, \varkappa) &= s^2[-\beta^2 - 2J_c + \beta J_s(3 + \beta^2)] + s[(14/3)\beta + \\
&+ \beta^3 + 2\beta J_c - \beta^2 J_s(3 + \beta^2)] - 3i\varkappa s^2[-\beta + J_s(1 + \beta^2)] - 2i\varkappa s[7/3 + 2\beta^2 + J_s - J_s\beta(3 + 2\beta^2)].
\end{aligned} \quad (2.3)$$

The corresponding expressions for $S_n(\alpha)$ are obtained from those of $R_n(\beta, \varkappa)$ if in formulas (2.3) we make the substitutions $\beta \to \alpha$, $\varkappa \to 1$, $J_s \to j_s$, and $J_c \to j_c$. The quantities $Q_n(\alpha, \beta, \varkappa)$ are expressed in the form

$$\begin{aligned}
Q_1 = Q_2 = Q_3 = 0,\ Q_4 &= \varkappa^2 s^2[1/(1 + \beta^2) - 3 + 3\beta J_s] + \\
&+ \varkappa^2 s[-2\beta/(1 + \beta^2) + 3\beta + J_s(1 - 3\beta^2)] + \varkappa^2[\beta^2/(1 + \beta^2) - \beta J_s] + \\
&+ (1/2)s[-3\alpha + j_s(1 + 3\alpha^2)] + [-1 + \alpha j_s]; \\
Q_5 &= \varkappa^2 s^2[1 - \beta J_s] + \varkappa^2 s[-\beta + \beta^2 J_s] + (1/2)s[\alpha - j_s(1 + \alpha^2)].
\end{aligned} \quad (2.4)$$

The variables α and β are complex. Their real and imaginary parts are given by formulas (1.11). Using these formulas we change over to variables s and $\varkappa$ and separate the real and imaginary parts in functions J_s and j_s:

$$\begin{aligned}
J_s &= (1/2)\mathrm{arctg}[2s/(s^2 + \varkappa^2 - 1)] + i(1/4)\ln\{[s^2 + (\varkappa - 1)^2]^2/[s^4 + 2(\varkappa^2 + 1)s^2 + (\varkappa^2 - 1)^2]\}; \\
j_s &= (1/2)\mathrm{arctg}(2/s) - i(1/4)\ln(1 + 4/s^2).
\end{aligned} \quad (2.5)$$

We now substitute (1.11), (2.5) into (2.2)-(2.4). This enables us to separate the real and the imaginary parts of function F_n:

$$F_n = a_n + ib_n, \quad (2.6)$$

where the coefficients a_n and b_n are given by the formulas

$$\rho c^2 a_1 = (5/16)[14(\varkappa^2 - 1)s^4 + (-23 + 16\varkappa^2 + 7\varkappa^4)s^2] - (s^2 + 5)/(s^2 + 4) + P_1 - P_2P_3 + P_4P_5;$$

$$\rho c^2 a_2 = (1/16)[10(1-\varkappa^2)s^4 + (13 - 8\varkappa^2 - 5\varkappa^4)s^2] + P_2P_6 - P_4P_7;$$
$$\rho c^2 a_3 = (1/16)[2(\varkappa^2-1)s^4 + (\varkappa^4-1)s^2] - P_2P_8 + P_4P_9;$$
$$\rho c^2 a_4 = -1{,}5\varkappa^2 s^2 - P_1 + P_2P_{10}; \qquad \rho c^2 a_5 = 0{,}5\varkappa^2 s^2 - P_2P_{11};$$
$$\rho c^2 b_1 = (5/48)[-21P_0 + (76 - 21\varkappa - 34\varkappa^3 - 21\varkappa^5)s] - P_{12} + 2/s(s^2+4) - P_{13}P_3 - P_{14}P_5; \tag{2.7}$$
$$\rho c^2 b_2 = (1/48)[15P_0 + (-44 + 15\varkappa + 14\varkappa^3 + 15\varkappa^5)s] + P_{13}P_6 + P_{14}P_7;$$
$$\rho c^2 b_3 = (1/48)[-3P_0 + (4 - 3\varkappa + 2\varkappa^3 - 3\varkappa^5)s] - P_{13}P_8 - P_{14}P_9;$$
$$\rho^2 c b_4 = 1{,}5\varkappa^3 s + P_{12} + P_{13}P_{16}; \quad \rho c^2 b_5 = -0{,}5\varkappa^3 s - P_{13}P_{11}.$$

The quantities P_n are of the form

$$P_0 = (\varkappa - 1)s^5 + 2(-2 + \varkappa + \varkappa^3)s^3;$$
$$P_1 = \{\varkappa^2[s^4 + (2+3\varkappa^2)s^2 + (1-\varkappa^2)]/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2]\};$$
$$P_2 = (1/2)\operatorname{arctg}[2s/(s^2+\varkappa^2-1)]; \; P_3 = (5/16)[7s^7 +$$
$$+ 21(1+\varkappa^2)s^5 + 3(7 + 10\varkappa^2 + 7\varkappa^4)s^3 + (7 + 9\varkappa^2 + 9\varkappa^4 + 7\varkappa^6)s];$$
$$P_4 = (1/2)\operatorname{arctg}(2/5); \; P_5 = (5/16)[7s^7 + 42s^5 + 72s^3 + 32s];$$
$$P_6 = (1/16)[5s^7 + 15(1+\varkappa^2)s^5 + 3(5 + 6\varkappa^2 + 5\varkappa^4)s^3 + (5 + 3\varkappa^2 +$$
$$+ 3\varkappa^4 + 5\varkappa^6)s]; \; P_7 = (1/16)[5s^7 + 30s^5 + 48s^3 + 16s]; \; P_8 = (1/16)[s^7 + \tag{2.8}$$
$$+ 3(1+\varkappa^2)s^5 + (3 + 2\varkappa^2 + 3\varkappa^4)s^3 + (1 - \varkappa^2 - \varkappa^4 + \varkappa^6)s]; \; P_9 =$$
$$= (1/16)[s^7 + 6s^5 + 8s^3]; \; P_{10} = 1{,}5\,\varkappa^2 s[s^2 + (\varkappa^2+1)]; \; P_{11} =$$
$$= 0{,}5\varkappa^2 s[s^2 + (\varkappa^2+1)]; \; P_{12} = 2\varkappa^5 s/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2];$$
$$P_{13} = (1/4)\ln\{[s^2 + (\varkappa-1)^2]^2/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2];$$
$$P_{14} = (1/4)\ln(1 + 4/s^2).$$

It is evident from the obtained results that after appropriate algebraic manipulations associated with the evaluation of integral I_{pqrs} the large parameters δ^n and J_c occurring in (1.12) mutually cancel out; this avoids the need of making the limiting transition $1/\delta \to 0$ in the final formulas.

3. Let us evaluate integral (1.2) for the transverse waves. For this purpose we must substitute the explicit values of functions g and h in accordance with (1.5) and of integrals J_{pqrs} and J_{pqr} in accordance with (1.9) into the expressions (1.8). Another simpler method consists in using formulas (1.7)-(1.9) which allow us to use the final formulas (2.3), (2.4) for the longitudinal waves.

The quantities (computed at this point) a_n, b_n, etc., and also variables c, q, $\varkappa$, s refer to the transverse waves and therefore must have the index t. As in § 2, for the sake of simplicity, we will drop index t in these quantities.

As before, integral I_{pqrs} is determined by expressions (2.1), (2.6); however, now coefficients a_n and b_n will be written in the form

$$\rho c^2 a_1 = (5/16)[14(1-\varkappa^2)s^4 + (23 - 16\varkappa^2 - 7\varkappa^4)s^2] -$$
$$- \varkappa^2[s^4 + (2+3\varkappa^2)s^2 + (1-\varkappa^2)]/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2] + u_1 + u_2u_3 - u_4u_5;$$
$$\rho c^2 a_2 = (1/16)[10(\varkappa^2-1)s^4 + (-13 + 8\varkappa^2 + 5\varkappa^4)s^2] - u_2u_6 + u_4u_7;$$
$$\rho c^2 a_3 = (1/16)[2(1-\varkappa^2)s^4 + (1-\varkappa^4)s^2] + u_2u_8 - u_4u_9;$$
$$\rho c^2 a_4 = -1{,}5s^2 - u_1 + u_4u_{10}; \; \rho c^2 a_5 = 0{,}5s^2 - u_4u_{11};$$
$$\rho c^2 b_1 = (5/48)[-21u_0 + (-76 + 21\varkappa + 34\varkappa^3 + 21\varkappa^5)s + \tag{3.1}$$
$$+ 2\varkappa^5 s/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2] - u_{12} + u_{13}u_3 + u_{14}u_5;$$
$$\rho c^2 b_2 = (1/48)[15u_0 + (44 - 15\varkappa - 14\varkappa^3 - 15\varkappa^5)s] - u_{13}u_6 - u_{14}u_7;$$
$$\rho c^2 b_3 = (1/48)[-3u_0 + (-4 + 3\varkappa - 2\varkappa^3 + 3\varkappa^5)s] + u_{13}u_8 + u_{14}u_9;$$
$$\rho c^2 b_4 = 1{,}5s + u_{12} - u_{14}u_{10}; \; \rho c^2 b_5 = -0{,}5s + u_{14}u_{11}.$$

Here the quantities u_n are of the form

$$u_0 = (1-\varkappa)s^5 + 2(2 - \varkappa - \varkappa^3)s^3; \; u_1 = (s^2+5)/(s^2+4);$$
$$u_2 = (1/2)\operatorname{arctg}[2s/(s^2 + \varkappa^2 - 1];$$
$$u_3 = (5/16)[7s^7 + 21(1+\varkappa^2)s^5 + 3(7 + 10\varkappa^2 + 7\varkappa^4)s^3 + (7 + 9\varkappa^2 + 9\varkappa^4 + 7\varkappa^6)s];$$
$$u_4 = (1/2)\operatorname{arctg}(2/s); \; u_5 = (5/16)[7s^7 + 42s^5 + 72s^3 + 32s]; \; u_6 =$$
$$= (1/16)[5s^7 + 15(1+\varkappa^2)s^5 + 3(5 + 6\varkappa^2 + 5\varkappa^4)s^3 + (5 + 3\varkappa^2 + 3\varkappa^4 + 5\varkappa^6)s]; \tag{3.2}$$

$$u_7 = (1/16)[5s^7 + 30s^5 + 48s^3 + 16s];\ u_8 = (1/16)[s^7 + 3(1 + \varkappa^2)s^5 + $$
$$+ (3 + 2\varkappa^2 + 3\varkappa^4)s^3 + (1 - \varkappa^2 - \varkappa^4 + \varkappa^6)s];\ u_9 = (1/16)[s^7 + 6s^5 + 8s^3];$$
$$u_{10} = 1.5s(s^2 + 2);\ u_{11} = 0{,}5s(s^2 + 2);\ u_{12} = 2/s(s^2 + 4);$$
$$u_{13} = (1/4)\ln\{[s^2 + (\varkappa - 1)^2]^2/[s^4 + 2(\varkappa^2 + 1)s^2 + (\varkappa^2 - 1)^2]\};$$
$$u_{14} = (1/4)\ln(1 + 4/s^2).$$

4. In order to compute the scattering coefficients of longitudinal and transverse waves and their propagation velocities from the known integrals I_{pqrs} we make use of the formula [2]

$$C_{il} = A_{ikpq}^{rslm} I_{pqrs} l_{km}.$$

Substituting (2.1), (2.6) into this formula we get

$$C_{il} = (a_1 + ib_1)\, l_{kmpqrs} A_{ikpq}^{rslm} + (a_2 + ib_2)(4l_{kmrq} A_{ikpq}^{rplm} + 2l_{kmpq} A_{ikpq}^{sslm}) +$$
$$+ (a_3 + ib_3)(l_{km} A_{ikpp}^{sslm} + 2l_{km} A_{ikpq}^{pqlm}) + (a_4 + ib_4)\, l_{kmrq} A_{ikpq}^{rplm} + (a_5 + ib_5)\, l_{km} A_{ikpq}^{pqlm}.$$

Depending on which of the waves are being considered, quantities a_n and b_n will be given by formulas (2.7), (2.8) or (3.1), (3.2). The contractions of the tensor A_{ikpq}^{rslm} for orthorhombic symmetry are well known [3]. Substituting their explicit value, we obtain

$$\begin{gathered}
C_{il} = (\lambda^* + \mu^*) l_{il} + \mu^* \delta_{il};\ \lambda^* = \lambda_1 + i\lambda_2;\ \mu^* = \mu_1 + i\mu_2;\\
\lambda_1 + \mu_1 = A_9 a_1 + 2(2A_5 + A_1)a_2 + (A_3 + 2A_7)a_3 + A_5 a_4 + A_7 a_5;\\
\lambda_2 + \mu_2 = A_9 b_1 + 2(2A_5 + A_1)b_2 + (A_3 + 2A_7)b_3 + A_5 b_4 + A_7 b_5;\\
\mu_1 = A_{10} a_1 + 2(2A_6 + A_2)a_2 + (A_4 + 2A_8)a_3 + A_6 a_4 + A_8 a_5;\\
\mu_2 = A_{10} b_1 + 2(2A_6 + A_2)b_2 + (A_4 + 2A_8)b_3 + A_6 b_4 + A_8 b_5.
\end{gathered} \tag{4.1}$$

The scattering coefficients of the waves and their velocities are found from formulas similar to those given in [2, 3]:

$$\begin{gathered}
\gamma_t(s_t) = \frac{\mu_2^t(s_t)}{2\rho a s_t c_t^2};\quad \gamma_l(s_l) = \frac{\lambda_2^l(s_l) + 2\mu_2^l(s_l)}{2\rho a s_l c_l^2};\\
v_t(s_t) = c_t + \Delta c_t;\quad v_l(s_l) = c_l + \Delta c_l;\quad 2\rho c_t \Delta c_t \equiv \mu_1^t(s_t) - s_t \frac{d\mu_1^t(s_t)}{ds_t};\\
2\rho c_l \Delta c_l \equiv \lambda_1^l(s_l) + 2\mu_1^l(s_l) - s_l \frac{d}{ds_l}\left[\lambda_1^l(s_l) - 2\mu_1^l(s_l)\right].
\end{gathered} \tag{4.2}$$

Substituting λ_i and μ_i from (4.1) into (4.2), we get

$$\begin{gathered}
\mu_2^t(s_t) = H_1 b_1^t(s_t) + H_2 b_2^t(s_t) + H_3 b_3^t(s_t) + H_4 b_4^t(s_t) + H_5 b_5^t(s_t);\\
\lambda_2^l(s_l) + 2\mu_2^l(s_l) = H_6 b_1^l(s_l) + H_7 b_2^l(s_l) + H_8 b_3^l(s_l) + H_9 b_4^l(s_l) + H_{10} b_5^l(s_l);\\
2\rho c_t \Delta c_t = H_1\left[a_1^t(s_t) - s_t \frac{da_1^t(s_t)}{ds_t}\right] + H_2\left[a_2^t(s_t) - s_t \frac{da_2^t(s_t)}{ds_t}\right] +\\
+ H_3\left[a_3^t(s_t) - s_t \frac{da_3^t(s_t)}{ds_t}\right] + H_4\left[a_4^t(s_t) - s_t \frac{da_4^t(s_t)}{ds_t}\right] + H_5\left[a_5^t(s_t) - s_t \frac{da_5^t(s_t)}{ds_t}\right];\\
2\rho c_l \Delta c_l = H_6\left[a_1^l(s_l) - s_l \frac{da_1^l(s_l)}{ds_l}\right] + H_7\left[a_2^l(s_l) - s_l \frac{da_2^l(s_l)}{ds_l}\right] +\\
+ H_8\left[a_3^l(s_l) - s_l \frac{da_3^l(s_l)}{ds_l}\right] + H_9\left[a_4^l(s_l) - s_l \frac{da_4^l(s_l)}{ds_l}\right] + H_{10}\left[a_5^l(s_l) - s_l \frac{da_5^l(s_l)}{ds_l}\right].
\end{gathered} \tag{4.3}$$

Here we have introduced the following notation:

$$\begin{gathered}
H_1 = A_{10} = B_{13};\ H_2 = 2(2A_6 + A_2) = (2/3)(B_7 + 3B_3);\\
H_3 = A_4 + 2A_8 = 3B_3;\ H_4 = A_6 = B_{14};\ H_5 = A_8 = (3/5)(2B_4 + B_3);\\
H_6 = A_9 + A_{10} = B_9;\\
H_7 = 2(2A_5 + 2A_6 + A_1 + A_2) = (4/9)(9B_{11} - 24B_{14} + 4B_7 + 12B_3);\\
H_8 = A_3 + A_4 + 2A_7 + 2A_8 = 3B_1;\ H_9 = A_5 + A_6 = B_{11};\\
H_{10} = A_7 + A_8 = (1/10)(15B_1 - 12B_3 + 16B_4),
\end{gathered} \tag{4.4}$$

where the coefficients A_n and B_n are expressed in terms of two-index elasticity constants with the use of formulas given in [3]. It is evident from the formulas obtained above that for computing the scattering coefficient γ_l of longitudinal waves we must use the second of formulas (4.2) in which the numerator is expressed in terms of the known coefficients B_n with the use of the second of formulas (4.3) and (4.4) and in terms of functions b_n with the use of formulas (2.7), (2.8). In changing over from parameter s to the wave number q we must make use of the formula $s_l = 1/q_{la}$ which follows from (1.3). The quantity $\varkappa_l$ is found with the use of the first formula in (1.6). Similarly, the scattering coefficient γ_t of the transverse waves is computed from formulas (4.2)-(4.4), (3.1), (3.2), (1.3), and the last of the formulas in (1.6).

5. In order to compute the frequency dependence of the velocity of propagation of the longitudinal waves we make use of the fourth formula in (4.2) and (4.3). Formulas (2.7), (2.8) enable us to obtain the following expressions:

$$\begin{gathered}\rho c^2(a_1 - s da_1/ds) = (5/16)[42(1-\varkappa^2)s^4 + (23 - 16\varkappa^2 - 7\varkappa^4)s^2] + \\ + (15/8)P_2 s^3 [7s^4 + 14(1+\varkappa^2)s^2 + (7 + 10\varkappa^2 + 7\varkappa^4)] - (15/8)P_4 s^3 (7s^4 + \\ + 28s^2 + 24) - (s^4 + 11s^2 + 20)/(s^2+4)^2 + P_{15} - P_{16}P_3 + P_{17}P_5; \\ \rho c^2(a_2 - s da_2/ds) = (1/16)[30(\varkappa^2 - 1)s^4 + (-13 + 8\varkappa^2 + 5\varkappa^4)s^2] - \\ - (3/8)P_2 s^3 [5s^4 + 10(1+\varkappa^2)s^2 + (5 + 6\varkappa^2 + 5\varkappa^4)] + (3/8)P_4 s^3 (5s^4 + 20s^2 + 16) + P_{16}P_6 - P_{17}P_7; \\ \rho c^2(a_3 - s da_3/ds) = (1/16)[6(1-\varkappa^2)s^4 + (1-\varkappa^4)s^2] + (1/8)P_2 s^3 [3s^4 + \\ + 6(1+\varkappa^2)s^2 + (3 + 2\varkappa^2 + 3\varkappa^4)] - (1/8)P_4 s^3 (3s^4 + 12s^2 + 8) - \\ - P_{16}P_8 + P_{17}P_9;\ \rho c^2(a_4 - s da_4/ds) = 1{,}5\varkappa^2 s^2 - 3P_2\varkappa^2 s^3 - P_{15} + \\ + P_{16}P_{10};\ \rho c^2(a_5 - s da_5/ds) = -0{,}5\varkappa^2 s^2 + P_2 \varkappa^2 s^3 - P_{16}P_{11},\end{gathered} \tag{5.1}$$

where P_n for n = 0-14 are given by expressions (2.8), while for n = 15-17 they are given by the formulas

$$\begin{gathered}P_{15} = \{\varkappa^2/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2]^2\}[s^8 + (4 + 7\varkappa^2)s^6 + \\ + (6 + 11\varkappa^2 + 3\varkappa^4)s^4 + (4 + \varkappa^2 - 2\varkappa^4 - 3\varkappa^6)s^2 + (1 - 3\varkappa^2 + 3\varkappa^4 - \varkappa^6)]; \\ P_{16} = s[s^2 - (\varkappa^2 - 1)]/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2];\ P_{17} = s/(s^2+4).\end{gathered} \tag{5.2}$$

Since in formulas (5.1), (5.2) all the quantities refer to longitudinal waves, they must have index l, which has been omitted for the sake of simplicity.

It follows from these results that the velocity of propagation v_l of longitudinal waves in polycrystals having orthorhombic or higher symmetry is given by formulas (4.2)-(4.4), (5.1), (5.2), (2.8), (1.4), and the first formula in (1.6).

The frequency dependence of the velocity of propagation of the transverse waves is determined similarly. From formulas (3.1), (3.2) we get

$$\begin{gathered}\rho c^2(a_1 - s da_1/ds) = (5/16)[42(\varkappa^2-1)s^4 + (-23 + 16\varkappa^2 + 7\varkappa^4)s^2] - \\ - (15/8)u_2 s^3 [7s^4 + 14(1+\varkappa^2)s^2 + (7 + 10\varkappa^2 + 7\varkappa^4)] + (15/8)u_4 s^3 (7s^4 + \\ + 28s^2 + 24) - \{\varkappa^2/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2]^2\}[s^8 + (4 + 7\varkappa^2)s^6 + \\ + (6 + 11\varkappa^2 + 3\varkappa^4)s^4 + (4 + \varkappa^2 - 2\varkappa^4 - 3\varkappa^6)s^2 + (1 - 3\varkappa^2 + 3\varkappa^4 - \varkappa^6)] + u_{15} + u_{16}u_3 - u_{17}u_5; \\ \rho c^2(a_2 - s da_2/ds) = (1/16)[30(1-\varkappa^2)s^4 + (13 - 8\varkappa^2 - 5\varkappa^4)s^2] + \\ + (3/8)u_2 s^3 [5s^4 + 10(1+\varkappa^2)s^2 + (5 + 6\varkappa^2 + 5\varkappa^4)] - (3/8)u_4 s^3 (5s^4 + 20s^2 + 16) - u_{16}u_6 + u_{17}u_7; \\ \rho c^2(a_3 - s da_3/ds) = (1/16)[6(\varkappa^2-1)s^4 + (\varkappa^4-1)s^2] - (1/8)u_2 s^3 [3s^4 + \\ + 6(1+\varkappa^2)s^2 + (3 + 2\varkappa^2 + 3\varkappa^4)] + (1/8)u_4 s^3 (3s^4 + 12s^2 + 8) + u_{16}u_8 - u_{17}u_9; \\ \rho c^2(a_4 - s da_4/ds) = 1{,}5s^2 - 3u_4 s^3 - u_{15} + u_{17}u_{10}; \\ \rho c^2(a_5 - s da_5/ds) = -0{,}5s^2 + u_4 s^3 - u_{17}u_{11},\end{gathered} \tag{5.3}$$

where u_n for n = 0-14 are given by expression (3.2), while for n = 15-17 they are given by the formula

$$u_{15} = (s^4 + 11s^2 + 20)/(s^2+4)^2;\ u_{16} = s[s^2 - (\varkappa^2-1)]/[s^4 + 2(\varkappa^2+1)s^2 + (\varkappa^2-1)^2];\ u_{17} = s/(s^2+4). \tag{5.4}$$

Formulas (5.3), (5.4) pertain to transverse waves and therefore all the quantities occurring in these formulas must have the index t; again, for the sake of simplicity, this index is omitted. We stress that the introduction of the corresponding index is obligatory ($\varkappa_l = 1/\varkappa_t = c_l/c_t$, $c_l \neq c_t$, $q_l \neq q_t$, $s_l \neq s_t$, etc.).

Thus, the frequency dependence of the velocity of propagation of transverse waves v_t in polycrystals with orthorhombic and higher symmetry is given by formulas (4.2)-(4.4), (5.3), (5.4), (3.2), (1.3), and (1.6).

It is evident from the formulas given above that the scattering coefficients and the velocities of ultrasonic waves in polycrystals have, generally, a complex frequency dependence. For matching the obtained results with the well-known formulas for low and high frequencies we carried out the limiting transition in expressions (2.1), (2.6)-(2.8), (3.1), and (3.2). Computations carried out by the method of series expansion in parameter $1/s$ showed that the asymptotic form of the long wave ($1/s = qa \ll 1$) completely coincides with the results of [3, 4]. In the asymptotic form of short waves ($1/s = qa \gg 1$) using the series expansion in s it is found that the correct expressions for I_{pqrs} are the following:

$$I^l_{pqrs} = \frac{l_q l_s}{\rho}\left[\frac{l_p l_r - \delta_{pr}}{c_t^2 - c_l^2} - \frac{5 l_p l_r}{4c_l^2} + i\frac{l_p l_r}{2c_l^2} a q_l\right];$$

$$I^t_{pqrs} = \frac{l_q l_s}{\rho}\left[5\frac{l_p l_r - \delta_{pr}}{4c_t^2} - \frac{l_p l_r}{c_l^2 - c_t^2} - i\frac{l_p l_r - \delta_{pr}}{2c_t^2} a q_t\right].$$

These expressions differ from the approximate formulas presented in [3] by a numerical factor which makes a significant contribution only in the velocity of propagation of short waves and does not change the scattering coefficient.

Using the formulas obtained above, the scattering coefficients of longitudinal and transverse ultrasonic waves were computed on a computer for aluminum for a wide range of wavelengths. The elastic constants of aluminum in units of 10^{11} dyn/cm^2 are as follows [5]: $c_{11} = 10.82$, $c_{12} = 6.13$, $c_{44} = 2.85$, and the density $\rho = 2.7$ g/cm^3. The correlation scale a is equal to the median radius of the representations r_{50} [1], i.e., the radius of such a grain for which 50% of the grain representations in the cross section of the cut have radius larger than r_{50} and 50% have smaller.

For the computations we took $a = 0.01$ cm. The results of the computations are given in Fig. 1, where the dashes denote the corresponding dependences computed from the known asymptotic formulas of [3] for cubic symmetry where $\langle a^3 \rangle = 8\pi a^3$; $\langle a \rangle = a$. The numbers 1, 2 denote the scattering coefficients $\gamma_t(s_t)$ and $\gamma_l(s_l)$ for the transverse and the longitudinal waves, respectively. The quantities $1/s_t$ for the transverse waves and $1/s_l$ for the longitudinal waves are plotted along the abscissa. Since $1/s = qa = \omega a/c$, the same graph describes also the frequency dependence of the attenuation coefficient. It is evident from the figure that the asymptotic formulas can be used if $qa \geq 10$ or $qa \leq 1/10$. The intersection of the curves γ_t and γ_l is in conformity with the known theoretical estimates by the asymptotic formulas according to which $\gamma_t > \gamma_l$ in the region of high frequencies, while $\gamma_t < \gamma_l$ in the region of low frequencies. The slope of the curves in the figure reflects the dependence $\gamma \sim \omega^4$ for low frequencies and $\gamma \sim \omega^2$ for high frequencies, which was also confirmed experimentally [1]. In the intermediate region where $aq \sim 1$, the slope of the curves for the transverse waves changes monotonically ensuring a smooth transition from the dependence $\gamma_t \sim \omega^4$ to $\gamma_t \sim \omega^2$. However, there is no such smooth transition for the longitudinal waves. The slope of the curve described by the dependence $\partial \ln \gamma / \partial \ln \omega$ at first decreases from 4 to 1, then again increases to 2.

The present study has been carried out in an approximation which takes into consideration only the paired correlation between the inhomogeneity elements. This approach presupposes smallness of spatial fluctuations of the modulus of elasticity tensor. For a numerical estimate of the magnitude of fluctuations the ratios of the coefficients $c_{mn}/\langle c_{mn}\rangle$ are given in Table 1 for a number of polycrystals of orthorhombic symmetry. The reference values of c_{mn} are taken from [6], while the mean values are computed from the formulas [7]

$$\langle c_{11}\rangle = \langle c_{12} + 2c_{44}\rangle;\ \langle c_{12}\rangle = \langle k - (2/3)\mu\rangle;\ \langle c_{44}\rangle = \langle \mu \rangle;$$

$$\langle K \rangle = (1/9)[(c_{11} + c_{22} + c_{33}) + 2(c_{12} + c_{23} + c_{13})];$$

$$\langle \mu \rangle = (1/15)[(c_{11} + c_{22} + c_{33}) - (c_{12} + c_{23} + c_{13}) + 3(c_{44} + c_{55} + c_{66})].$$

It is evident that for most of these materials, the condition of smallness of the spatial fluctuations is satisfied ($c_{mn}/\langle c_{mn}\rangle \simeq 1$).

As an example of materials with large anisotropy we have shown argonite and $SrSO_4$, for which the approximation used here can lead to a large error.

LITERATURE CITED

1. E. P. Papadakis, "Ultrasonic attenuation caused by scattering in polycrystalline media," in: Physical Acoustics (edited by W. P. Mason), Vol. 4B, Academic Press (1967).
2. E. M. Lifshits and G. D. Parkhomovskii, "Theory of propagation of ultrasonic waves in polycrystals," Zh. Éksp. Teor. Fiz., 20, No. 2 (1950).
3. A. A. Usov, A. G. Fokin, and T. D. Shermergor, "Theory of propagation of ultrasonic waves in polycrystals," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1972).
4. A. B. Bhatia and B. A. Moore, "Scattering of high-frequency sound waves in polycrystalline materials," J. Acoust. Soc. Amer., 31, No. 8 (1959).
5. G. Khangtington, "Elastic constants of crystals," Usp. Fiz. Nauk, 74, No. 3 (1961).

6. O. L. Anderson, "Determination and some uses of isotropic elastic constants of polycrystalline aggregates using single crystal data," in: Physical Acoustics (edited by W. P. Mason), Vol. 3B, Academic Press (1965).
7. T. D. Shermergor, "Modulus of elasticity of inhomogeneous materials," in: Reinforcement of Metals by Fibers [in Russian], Nauka, Moscow (1973).

SOME LAWS FOR PRECIPITATION OF AEROSOLS ON A CHARGED COLLECTOR IN THE REYNOLDS NUMBER RANGE 10-100

V. V. Smirnov

UDC 633.6.011:621.359.4

Experimental data are presented on the efficiency of electrostatic precipitation of aqueous aerosol particles on a strongly charged sphere in the medium Reynolds number range (Re = 10-100). The asymptotic solutions for the problem are presented, and typical errors allowable in interpreting this type of experiment are discussed.

Existing theoretical and experimental data [1-6] on the efficiency of the electrostatic precipitation of aerosol particles on charged bodies of very simple shape refer mainly to cases where there is either viscous (Reynolds number much less than 1) or uniform flow (the electric forces significantly predominate over the hydrodynamic) of air carrying aerosols over the collector.

In a number of cases associated with filtration and elution of aerosols by precipitation particles and artificial bodies [7-10] so-called medium or intermediate Reynolds numbers (Re = 5-100) are achieved. For this situation information on the laws of electrostatic precipitation of particles at an obstacle is practically nonexistent.

The present work analyzes the results of measurements of capture coefficients of neutral and charged particles of aqueous aerosols by a fixed, charged spherical collector. The Reynolds numbers values based on the sphere diameter fall in the range 10-100.

The experimental technique and some of the initial measurement data have been described in [7-8]. The essence of the technique is as follows.

A one-dimensional jet of droplets of a given size and charge is generated in a flow of moist air, washing a metal sphere of diameter 0.4 cm. The sphere potential is varied in the range 0 to ± 6000 V, and the droplet charge from 0 to ± 100e, the droplet diameter from 10 to 30 μ, and the flow speed from 4 to 40 cm/sec. From analysis of television photographs of the limiting trajectories for the droplet motion near the spherical collector we determined the capture coefficient, defined as the ratio of the area of the stream tube of precipitated particles to the projected area of the sphere.

The results of the tests of interaction of uncharged conducting droplets with a charged sphere, with an electric field intensity on its surface of 5, 10, and 20 kV/cm, are shown in Fig. 1; the capture coefficient K is shown as a function of the dimensionless coagulation parameter β, which characterizes the ratio of the mirror and aerodynamic forces acting on the particle

$$\beta = (2U^2d^2/3\pi\eta D^3u_\infty)\cdot(\varepsilon - 1)/(\varepsilon + 2), \tag{1}$$

where d and ε are the size and dielectric constant of the droplets; U and D are the potential and diameter of the sphere; η is the dynamic viscosity for air; and u_∞ is the flow velocity at infinity. We note that for U = 0 quite low values of K < 0.05 have been observed [8].

Obninsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 190-193, May-June, 1976. Original article submitted May 11, 1975.